Assignment 4.

1. $x = -49.1^\circ$ or 130.9°.
2. (a) omit
   (b) 0.955 or 5.33
3. $x = 48.2^\circ$ or 311.8° or 120° or 240°
4. (a) $R = 13$, $\alpha = 22.6.2^\circ$
   (b) 17.1° or 297.7°
5. (a) omit
   (b) $x = \frac{\pi}{8}$ or $\frac{5}{8}\pi$

Bonus question:

1. (a) 0.894, 0.0599 or -0.835
   (b) $\pm\frac{11}{2}$.
2. $4 : 5 : 6$

As-9 Ethan

31 + 10/31 FANTASTIC!

Assignment 4.

1. Solve the equation $\sin(x - 30°) = 3\cos(x - 60°)$ for $-180° \leq x \leq 180°$. [5]

$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = 3\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$

$-\sqrt{3}\sin x - 2\cos x = 0$

$-\sqrt{3}\sin x = 2\cos x$

$\tan x = -\frac{2}{\sqrt{3}}$

$x = \tan^{-1}\left(-\frac{2}{\sqrt{3}}\right) + k\cdot 180°, k \in \mathbb{Z}$

hence $x = -49.1°$ or $130.9°$

2. (a) Prove the identity $\cos\left(x + \frac{1}{6}\pi\right) + \sin\left(x + \frac{1}{3}\pi\right) \equiv \sqrt{3}\cos x$. [3]

$LHS = \frac{\sqrt{3}}{2}\cos x - \cancel{\frac{1}{2}\sin x} + \cancel{\frac{1}{2}\sin x} + \frac{\sqrt{3}}{2}\cos x$

$= \sqrt{3}\cos x = RHS$.

(b) Hence solve the equation $\cos\left(x + \frac{1}{6}\pi\right) + \sin\left(x + \frac{1}{3}\pi\right) = 1$ for $0 < x < 2\pi$. [3]

$\sqrt{3}\cos x = 1$

$\cos x = \frac{1}{\sqrt{3}}$

$x = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) + 2k\pi$

or $-\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) + 2k\pi, k \in \mathbb{Z}$

$x = 0.955$ or 5.33

3. Solve the equation $\sec x = 4 - 2\tan^2 x$, giving all solutions in the interval $0° \leq x \leq 360°$. [6]

$\sec x = 6 - 2(1 + \tan^2 x)$

$\sec x = 6 - 2\sec^2 x$

$2\sec^2 x + \sec x - 6 = 0$

$(2\sec x - 3)(\sec x + 2) = 0$

$\sec x = \frac{3}{2}$ or -2

$\cos x = \frac{2}{3}$ or $-\frac{1}{2}$

$x = \pm\cos^{-1}\left(\frac{2}{3}\right) + k\cdot 360°$

or $\pm\cos^{-1}\left(-\frac{1}{2}\right) + k\cdot 360°, k \in \mathbb{Z}$

$x = 48.2°$ or $311.8°$ or $120°$ or $240°$

4. (a) Express $12\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

$12\cos\theta - 5\sin\theta$

$= 13\left(\frac{12}{13}\cos\theta - \frac{5}{13}\sin\theta\right)$

$= 13\cos\left(\theta + \tan^{-1}\left(\frac{5}{12}\right)\right)$

hence $R = 13$, $\alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$

(b) Hence solve the equation $12\cos\theta - 5\sin\theta = 10$, giving all solutions in the interval $0° \leq \theta \leq 360°$. [4]

$13\cos(\theta + \alpha) = 10$

$\cos(\theta + \alpha) = \frac{10}{13}$

$\theta + \alpha = \pm\cos^{-1}\left(\frac{10}{13}\right) + k\cdot 360°$

$\theta = \pm\cos^{-1}\left(\frac{10}{13}\right) - \tan^{-1}\left(\frac{5}{12}\right) + k\cdot 360°, k \in \mathbb{Z}$

$\theta = 17.1°$ or $297.7°$

5. (a) Prove the identity $\tan\left(x + \frac{1}{4}\pi\right) + \tan\left(x - \frac{1}{4}\pi\right) \equiv 2\tan 2x$. [4]

$$LHS = \frac{\tan x + 1}{1 - \tan x} + \frac{\tan x - 1}{1 + \tan x} = \frac{(1+\tan x)^2 - (1-\tan x)^2}{1 - \tan^2 x} = \frac{2 \cdot 2\tan x}{1 - \tan^2 x}$$

$$= 2\tan 2x = RHS.$$

(b) Hence solve the equation $\tan\left(x + \frac{1}{4}\pi\right) + \tan\left(x - \frac{1}{4}\pi\right) = 2$, for $0 \le x \le \pi$. [3]

$$2\tan 2x = 2$$

$$\tan 2x = 1$$

$$2x = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}.$$

$$x = \frac{\pi}{8} + \frac{1}{2}k\pi,\ k \in \mathbb{Z}$$

$$x = \frac{\pi}{8} \text{ or } \frac{5}{8}\pi.$$

Total mark of this assignment: 31.

$\frac{1}{3}\tan^{-1} y = \cos^{-1}\frac{2}{\sqrt{5}}$

(†) Bonus questions:

1. Show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

Given that $\theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and that θ is acute, show that $\tan 3\theta = \frac{11}{2}$.
Hence find all solutions of the equations.

(a) $\tan\left(3\cos^{-1} x\right) = \frac{11}{2}$,

(b) $\cos\left(\frac{1}{3}\tan^{-1} y\right) = \frac{2}{\sqrt{5}}$.

1. $$\tan 3\theta = \cancel{\tan\theta}\ \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta}$$

$$= \frac{\tan\theta + \frac{2\tan\theta}{1-\tan^2\theta}}{1 - \tan\theta \cdot \frac{2\tan\theta}{1-\tan^2\theta}} = \frac{3\tan\theta - \tan^3\theta}{1 - \tan^2\theta - 2\tan^2\theta}$$

$$= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\tan\theta = \frac{1}{2}$$

$$\tan 3\theta = \frac{3 \times \frac{1}{2} - \left(\frac{1}{2}\right)^3}{1 - 3 \times \left(\frac{1}{2}\right)^2} = \frac{11}{2}$$

~~(a) $\cos^{-1}(x) = \cos^{-1}x = \theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$~~

~~$x = \frac{2}{\sqrt{5}}$~~

~~(b) $\frac{1}{3}\tan^{-1} y = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$~~

~~$\frac{1}{3}\tan^{-1} y = \pm\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$~~

(a) $3\cos^{-1} x = \tan^{-1}\left(\frac{11}{2}\right) + k\pi,\ k \in \{0, 1, 2\}$ ~~$\{-1, 0, 1\}$~~.

$x = \cos\left(\frac{1}{3}\tan^{-1}\left(\frac{11}{2}\right) + \frac{1}{3}k\pi\right),\ k \in \{0, 1, 2\}$ ~~$\{-1, 0, 1\}$~~

$x =$ ~~0.835 or~~ 0.894 or ~~0.5 0.060~~ 0.0599.
or -0.835

(b). $\frac{1}{3}\tan^{-1} y = \pm\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

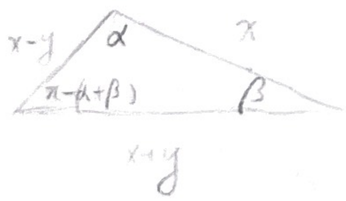
$y = \tan\left(\pm 3\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$

$y =$ ~~$\frac{11}{2}$ or~~ $\pm\frac{11}{2}$

2. The sides of a triangle have lengths $x - y$, x and $x + y$, where $x > y > 0$. The largest and smallest angles of the triangle are α and β, respectively. Show that

$$4(1 - \cos\alpha)(1 - \cos\beta) = \cos\alpha + \cos\beta.$$

In the case $\alpha = 2\beta$, show that $\cos\beta = \frac{3}{4}$ and hence find the ratio of the lengths of the sides of the triangle.



By using cosine law violently, we have the results easily.

$$4(1 - \cos 2\beta)(1 - \cos\beta) = \cos 2\beta + \cos\beta$$

$$8\cos^3\beta - 10\cos^2\beta - 9\cos\beta + 9 = 0.$$

$\cos\beta = -1$ (rejected), or $\frac{3}{2}$ (rejected), or $\frac{3}{4}$

~~$\cos\beta = \frac{3}{4}$~~

$$\frac{1}{2}x(x-y)\sin\alpha = \frac{1}{2}x(x+y)\sin\beta$$

$$\frac{2\sin\beta\cos\beta}{\sin\beta} = \frac{x+y}{x-y}$$

$$\frac{x+y}{x-y} = \frac{3}{2}$$

hence [illegible]

Take this part back home and work on these for fun!

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